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Combined mass and heat transfer during nonisothermal absorption in gas–liquid slug flow with small bubbles in liquid plugs

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Abstract

A model for combined mass and heat transfer during nonisothermal absorption of a one component gas in a vertical gas–liquid slug flow with liquid plugs containing small bubbles is suggested. Under the assumptions of a perfect mixing of the dissolved gas in liquid plugs and uniform temperature distribution in liquid plugs, recurrent relations for the dissolved gas concentration and temperature in the *n*th liquid plug and mass and heat fluxes from the *n*th unit cell of a gas–liquid slug flow are derived. The total mass and heat fluxes in a gas–liquid slug flow are determined. Theoretical results are compared with the available experimental data. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

a specific interphase area per unit mixture volume [m^{-1}]
a_L thermal diffusivity of liquid [m^2 s^{-1}]
A cross-section area of a channel [m^2]
b coefficient in equation (13) [kg m^{-3}]
c concentration of a soluble gas [kg m^{-3}]
c_e concentration of a soluble gas at exit [kg m^{-3}]
c'_0 equilibrium concentration at initial temperature [kg m^{-3}]
c_s concentration of a soluble gas at gas–liquid interphase [kg m^{-3}]
c_p specific heat [kJ kg^{-1} K^{-1}]
d coefficient in equation (13) [kg K^{-1} m^{-3}]
d_b diameter of a spherical bubble [m]
d_d diameter of a droplet [m]
d_{ch} channel diameter [m]
D coefficient of molecular diffusion in a liquid phase [m^2 s^{-1}]
D_G coefficient of molecular diffusion in a gas phase [m^2 s^{-1}]
$Fo = Dt/d_d^2$ Fourier number
g acceleration of gravity [m s^{-2}]
H heat of absorption [kJ kg^{-1}]
k number of spherical bubbles in a liquid plug
k_L liquid-side mass transfer coefficient [m s^{-1}]
$K = c_p\rho/d \cdot H$ dimensionless number
L_B length of a Taylor bubble [m]
L_c length of a unit cell of gas–liquid slug flow [m]
L_L length of a liquid plug [m]
L_T length of a tube [m]
Le Lewis number, D/a_L
$M = (W_T/c_p\rho + W_c)/Q_L$ dimensionless number
N number of unit cells of slug flow
$Pe = Ud_{ch}/D_G$ Peclet number
q_c mass flux density [kg m^{-2} s^{-1}]
q_T heat flux density [kJ m^{-2} s^{-1}]
Q_c mass flux [kg s^{-1}]
Q_L liquid flux [m^3 s^{-1}]
Q_T heat flux [kJ s^{-1}]
r_b spherical bubble radius [m]
R tube radius [m]
S surface area of a Taylor bubble [m^2]
S_b surface area of a spherical bubble [m^2]
t time [s]
T temperature of a liquid [K]
T'_0 equilibrium temperature at initial concentration [K]

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U_∞ velocity of a Taylor bubble rising in a stagnant fluid [m s^{-1}]
U_b velocity of a small spherical gas bubble rising in a stagnant fluid [m s^{-1}]
U_{GB} Taylor bubble velocity [m s^{-1}]
$\bar{U}_{GS}$ bubble swarm velocity [m s^{-1}]
U_G^s gas superficial velocity [m s^{-1}]
U_L^s liquid superficial velocity [m s^{-1}]
$\bar{U}_{LS}$ average liquid velocity in the plug [m s^{-1}]
U_M^s mixture superficial velocity [m s^{-1}]
v_r, v_θ velocity components [m s^{-1}]
$W_c = \beta S + \beta_b k S_b$ [m^3 s^{-1}]
$W_T = \alpha S + \alpha_b k S_b$ [kJ s^{-1} K^{-1}]
y, z coordinates [m].

Greek symbols
α coefficient of heat transfer for a single Taylor bubble [kJ m^{-2} s^{-1} K^{-1}]
α_b coefficient of heat transfer for a single spherical bubble [kJ m^{-2} s^{-1} K^{-1}]
β coefficient of mass transfer for a single Taylor bubble [m s^{-1}]
β_b coefficient of mass transfer for a single spherical bubble [m s^{-1}]
δ thickness of a liquid film [m]
λ thermal conductivity of liquid [kJ m^{-1} s^{-1} K^{-1}]
ν kinematic viscosity [m^2 s^{-1}]
$\xi(L_B/d_{ch})$ shape factor for a Taylor bubble
ρ liquid density [kg m^{-3}]
ρ_G gas density [kg m^{-3}]
σ surface tension [N m^{-1}]
φ_S cross-sectional average void fraction in liquid plug
φ_B Taylor bubble cross-sectional average void fraction.

Subscripts
b bubble
e value at exit
G gas
L liquid
s value at the interface
∞ value at infinity
0 value at the inlet.

1. Introduction

The rates of mass transfer during gas absorption by liquid in a slug flow are significantly higher than those in film or bubbly flows (see, e.g., Ref. [1]). High rates of mass transfer in slug flow are attained due to a complete destruction of the concentration boundary layer at the trailing edge of a slug by a vortex in a liquid plug (see, e.g., Campos and Guedes de Carvalho [2], Dukler, Moalem Maron and Brauner [3]). In film absorbers a destruction of a concentration boundary layer is achieved by employing a multistage absorption in agitated vessels (see, e.g., Ramm [4]). There are also other reasons for the high mass transfer rates in gas–liquid slug flow, like a high level of turbulence in a liquid caused by a vortex at the trailing edge of a slug in a liquid plug (Campos and Guedes de Carvalho [2], Moalem Maron et al. [5]), presence of small gas bubbles in liquid plugs (Fernandes et al. [6]) which increase the gas–liquid contact surface area.

Results of the investigation of mass and heat transfer during nonisothermal absorption in gas–liquid slug flow can be applied for determining rates of heat and mass transfer in absorbers of solar absorption refrigerators and absorbers in chemical engineering. The present investigation generalizes the results of Elperin and Fominykh [7, 8] on nonisothermal gas absorption from a linear cluster of slugs without small bubbles in the liquid plug for a case of slug flow with small bubbles in liquid plugs.

2. State-of-the art in nonisothermal absorption

Combined heat and mass transfer during nonisothermal absorption were investigated for gas absorption by: laminar smooth films, wavy liquid films, turbulent films, droplets and bubbles. Nonisothermal gas absorption by a laminar liquid film in the approximation of the uniform velocity across the film was investigated in [9–11]. Nonisothermal film absorption was studied by Grossman [12] and Yang and Wood [13] in a case of a semi-parabolic velocity profile in a liquid film, and a laminar model was extended to the turbulent flow conditions in [14]. Combined heat and mass transfer during nonisothermal absorption of vapor into a falling liquid film for a case of comparable concentration levels of absorbate and absorbent was studied theoretically in [15] and experimentally in [16, 17]. Yang and Jou [18] investigated numerically gas absorption with heat release in the approximation of an infinite dilution of the absorbate by a wavy liquid film. It is noted in [18] that wave formation on the surface of liquid film causes an increase in the rate of heat and mass transfer. Influence of wave formation at gas–liquid interface on the rate of heat and mass transfer in film flows was discussed also in [19, 20]. Combined mass and heat transfer during falling film absorption on a vertical cylindrical tube, on horizontal cylindrical tube and on a fluted tube was investigated [21–24]. Combined mass- and heat transfer by stationary spherical droplets was investigated analytically by Nakoryakov and Grigorieva [9] using a system of nonstationary heat and mass transfer equations. They showed that heat and mass transfer inside a droplet is determined by the dimensionless numbers K, Le, Fo and by the concentration driving force $dT_0 + b - c_0$. Morioka et al. [25] investigated numerically combined heat and mass transfer during pure vapor absorption by a moving droplet of an aqueous solution of LiBr. This analysis showed that temperature and concentration distributions in a droplet are strongly affected by the internal circulation which is caused by the shear forces at steam–liquid interface. The absorption rate increases by about several tens percent compared with that for the stationary droplet due to the internal flow in the droplet caused by external vapor flow. Combined heat and mass transfer in bubbly liquids was investigated in Refs. [26–31].

3. Combined mass and heat transfer from a single slug

Mass and heat transfer during absorption of a pure soluble gas from a short rising slug in a tube accompanied by a heat release was investigated by Elperin and Fominykh [7, 8] in the approximation of an infinite dilution of an absorbate. The thermodynamic parameters were assumed to be constant, and only the resistance to mass and heat transfer in the liquid phase was taken into account. It was assumed that the heat released during absorption was dissipated in the liquid phase and mass and heat transfer did not affect the hydrodynamics of the liquid phase. The equilibrium condition at the gas–liquid interface is described by the linear dependence of the concentration on temperature. Development of a thin diffusion and temperature boundary layers in liquid starts from the leading edge of a slug. Convective diffusion and heat transfer are determined by fluid velocity at the slug's surface. Fluid velocity at the interface can be determined from the Bernoulli equation and is equal to $u_s = \sqrt{2gz}$ (see [32]). The slug–liquid interface is assumed to be a surface of revolution obtained by the rotation of a curve $y(z)$ around a z-axis (see Fig. 1). The equation of a curve $y(z)$ was derived by Davis and Taylor [32]. Elperin and Fominykh [7, 8] derived the following equations for averaged mass and heat transfer coefficients, multiplied on slugs surface area:

$$\beta S = \frac{Q_c}{c'_o - c_0} = \frac{4(\pi D)^{1/2}(g/d_{ch})^{1/4} d_{ch}^2 \xi(L_B/d_{ch})}{1-\sqrt{Le/K}} \tag{1}$$

$$\alpha S = \frac{Q_T}{T'_0 - T_0} = \frac{4\pi^{1/2}\lambda a_L^{-1/2}(g/d_{ch})^{1/4} d_{ch}^2 \xi(L_B/d_{ch})}{1-K/\sqrt{Le}} \tag{2}$$

where $Le = D/a_L$, $K = c_p\rho/d \cdot H$, $c'_0 = dT_0 + b$ is the equilibrium concentration corresponding to the initial temperature, $T'_0 = (c_0 - b)/d$ the equilibrium temperature corresponding to the initial concentration, H the heat of absorption, d the coefficient in the linear dependence between concentration and temperature at gas–liquid interface,

$$\xi\left(\frac{L_B}{d_{ch}}\right) = \left(\int_0^{L_B/d_{ch}} \left(\frac{y(z)}{d_{ch}}\right)^2 \sqrt{1+\left(\frac{dy(z)/d_{ch}}{dz/d_{ch}}\right)^2} \sqrt{\frac{2gz}{d_{ch}}}\, d\left(\frac{z}{d_{ch}}\right)\right)^{1/2}. \tag{3}$$

$\xi(L_B/d_{ch})$ is a shape factor of a Taylor bubble, introduced in [33]. For the case without heat release ($H = 0$ and $K \to \infty$) equation (1) recovers the formula for isothermal absorption derived by Elperin and Fominykh [34, 35]. For long gas slugs coefficients of mass and heat transfer can be determined from the theory of combined mass and heat transfer in the falling liquid films [9]:

$$\alpha = \frac{q_T}{(T'_0 - T_0)} = \frac{2\lambda\sqrt{u/(a_L L_B)}}{\sqrt{\pi}(1-K/\sqrt{Le})} \tag{4}$$

$$\beta = \frac{q_c}{(c'_0 - c_0)} = \frac{2D\sqrt{u/(DL_B)}}{\sqrt{\pi}(1-\sqrt{Le/K})} \tag{5}$$

where u is the liquid velocity at gas–liquid interface in a liquid film, falling between a body of a gas slug and a tube. For isothermal absorption equation (5) yields:

$$\beta = 2\sqrt{\frac{Du}{\pi L_B}}. \tag{6}$$

4. Combined mass and heat transfer from a single spherical bubble

The size of a small spherical bubble in a liquid plug was determined by Taitel et al. [36] as:

$$d_b = \left(\frac{0.4\sigma}{(\rho-\rho_G)g}\right)^{1/2}. \tag{7}$$

For SO_2 gas bubbles in water $d_b = 1.72$ mm. Rise velocity of a small spherical bubble in the presence of a bubble swarm is (see, e.g., Fernandes et al. [6]):

$$U_b = 1.53\left(\frac{\sigma g(\rho-\rho_G)}{\rho^2}\right)^{1/4}(1-\varphi_S)^{1/2} \tag{8}$$

where φ_S is a void fraction in a bubble swarm. For SO_2 gas bubbles in water when $\varphi_S = 0.25$, $U_b = 0.21$ m s^{-1}. The concentration distribution of the dissolved gas and temperature distribution in the liquid during non-isothermal absorption of the medium-size spherical bubbles ($0.1 < d_b < 2$ mm) can be written in the form of

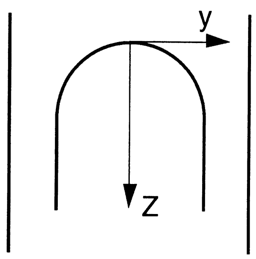

Fig. 1. Schematic view of a Taylor bubble.

equations of convective diffusion and energy:

$$v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r}\frac{\partial c}{\partial \theta} = D\frac{\partial^2 c}{\partial r^2} \tag{9}$$

$$v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} = a_L\frac{\partial^2 T}{\partial r^2} \tag{10}$$

with boundary conditions:

$$c = c_s, T = T_s \quad \text{at } r = r_b \tag{11}$$

$$c = c_0, T = T_0 \quad \text{at } r \to \infty \tag{12}$$

$$c = dT + b \quad \text{at } r = r_b \tag{13}$$

$$\lambda\frac{\partial T}{\partial r} = HD\frac{\partial c}{\partial r} \quad \text{at } r = r_b. \tag{14}$$

The velocity distribution near the bubble's surface at $r = r_b$ is determined by the following expressions (see, e.g., Levich [37]):

$$v_r^{(0)} = -3U_b\frac{y_1}{r_b}\cos\theta, \quad v_\theta^{(0)} = \frac{3}{2}U_b\left(1 - \frac{y_1}{r_b}\right)\sin\theta \tag{15}$$

where $y_1 = r - r_b$. Formulas (15) are derived in the assumption that $y_1 \ll r_b$. Substituting $v_r = v_r^{(0)}$ and $v_\theta = v_\theta^{(0)}$ in the convective diffusion and energy equations (9) and (10) and applying the methods suggested by Levich [37] and Nakoryakov and Grigorieva [9], one arrives at the following formulas for the mass and heat transfer coefficients multiplied by the surface area of a single spherical gas bubble of a diameter d_b rising in a liquid plug with velocity U_b:

$$\beta_b S_b = \frac{Q_c}{(c_0' - c_0)} = \frac{2d_b^2(\pi D U_b/d_b)^{1/2}}{1 - \sqrt{Le}/K} \tag{16}$$

$$\alpha_b S_b = \frac{Q_T}{(T_0' - T_0)} = \frac{2\lambda d_b^2(\pi U_b/a_L d_b)^{1/2}}{1 - K/\sqrt{Le}}. \tag{17}$$

For isothermal absorption equation (16) yields:

$$\beta_b S_b = \frac{Q_c}{(c_s - c_0)} = 2d_b^2(\pi D U_b/d_b)^{1/2}. \tag{18}$$

5. Mass and heat transfer in gas–liquid slug flow

Consider mass and heat transfer in a linear cluster of Taylor bubbles for the case of high Reynolds numbers assuming that slug flow is stable and that the lengths of all the Taylor bubbles and of all the liquid plugs are equal but the length of a liquid plug can differ from the length of a Taylor bubble (see Fig. 2). All Taylor bubbles are rising with a constant velocity in a vertical pipe separated by the regions of liquid containing small bubbles. The dispersed small bubbles are assumed to be confined to the region between the Taylor bubbles, thus moving with the Taylor bubble velocity. The bubble size is small enough to cause the bubbles to remain spherical and to prevent agglomeration. Assume that the liquid is perfectly mixed in a liquid plug by a vortex and that temperature and concentration distributions of a dissolved gas in each liquid plug are homogeneous. The first Taylor bubble enters a liquid with initial concentration c_0 and temperature T_0. Since the mass and heat transfer coefficients for a single Taylor bubble and a single spherical bubble are known, one can determine mass and heat fluxes from this first unit cell of a cluster of Taylor bubbles and concentration of the dissolved gas and temperature in the first liquid plug. Then, repeating this procedure, one can determine mass and heat fluxes from all subsequent unit cells and concentration of a dissolved gas and temperature in all liquid plugs. Formulas for mass and heat fluxes from the first unit cell of gas–liquid slug flow comprising the first Taylor bubble and the first liquid plug read:

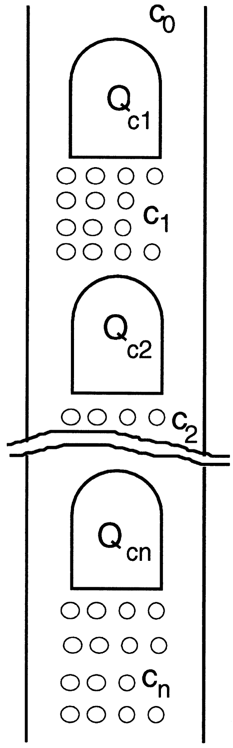

Fig. 2. Schematic view of a gas–liquid slug flow.

$$Q_{c1} = W_c(dT_0 + b - c_0) \tag{19}$$

$$Q_{T1} = W_T\left(\frac{c_0 - b}{d} - T_0\right) \tag{20}$$

where $W_c = \beta S + \beta_b k S_b$, $W_T = \alpha S + \alpha_b k S_b$, k is a number of small gas bubbles in the liquid plug. Taking into account that the total volume of small spherical bubbles in each liquid plug $V_G = \frac{1}{6}\pi d_b^3 k$ and liquid plug volume $V_L = (\pi_{ch}^2/4)\ L_L$, we derived a formula for a number of small spherical bubbles in a liquid plug:

$$k = \frac{3\varphi_S d_{ch}^2 L_L}{2d_b^3}. \quad (21)$$

The concentration of the dissolved gas and temperature in the first liquid plug following the first Taylor bubble reads:

$$c_1 = c_0 + \frac{Q_{c1}}{Q_L} = c_0 + \frac{W_c}{Q_L}(dT_0 + b - c_0), \quad (22)$$

$$T_1 = T_0 + \frac{Q_{T1}}{Q_L c_p \rho} = T_0 + \frac{W_T}{Q_L c_p \rho}\left(\frac{c_0 - b}{d} - T_0\right). \quad (23)$$

Similarly for the second unit cell and second liquid plug we obtain that

$$Q_{c2} = W_c(dT_1 + b - c_1) = W_c(c'_0 - c_0) - \frac{W_c}{Q_L}\left(W_c + \frac{W_T}{c_p\rho}\right)(c'_0 - c_0) \quad (24)$$

$$Q_{T2} = W_T\left(\frac{c_1 - b}{d} - T_1\right) = W_T(T'_0 - T_0) - \frac{W_T}{Q_L}\left(W_c + \frac{W_T}{c_p\rho}\right)(T'_0 - T_0) \quad (25)$$

$$c_2 = c_1 + \frac{Q_{c2}}{Q_L} = \frac{2W_c}{Q_L}(c'_0 - c_0) + c_0 - \frac{W_c}{Q_L^2}\left(W_c + \frac{W_T}{\rho c_p}\right)(c'_0 - c_0) \quad (26)$$

$$T_2 = T_1 + \frac{Q_{T2}}{\rho c_p Q_L} = \frac{2W_T}{Q_L c_p \rho}(T'_0 - T_0) + T_0 - \frac{W_T}{Q_L^2 c_p \rho}\left(W_c + \frac{W_T}{c_p\rho}\right)(T'_0 - T_0) \quad (27)$$

and for the third unit cell and third liquid plug:

$$Q_{c3} = W_c(dT_2 + b - c_2) = W_c(c'_0 - c_0) - \frac{2W_c}{Q_L}\left(W_c + \frac{W_T}{\rho c_p}\right)(c'_0 - c_0) + \frac{W_c}{Q_L^2}\left(W_c + \frac{W_T}{\rho c_p}\right)^2(c'_0 - c_0) \quad (28)$$

$$Q_{T3} = W_T\left(\frac{c_2 - b}{d} - T_2\right) = W_T(T'_0 - T_0) - \frac{2W_T}{Q_L}\left(W_c + \frac{W_T}{c_p\rho}\right)(T'_0 - T_0) + \frac{W_T}{Q_L^2}\left(W_c + \frac{W_T}{c_p\rho}\right)^2(T'_0 - T_0) \quad (29)$$

$$c_3 = c_2 + \frac{Q_{c3}}{Q_L} = \frac{3W_c}{Q_L}(c'_0 - c_0) + c_0 - \frac{3W_c}{Q_L^2}\left(W_c + \frac{W_T}{\rho c_p}\right)(c'_0 - c_0) + \frac{W_c}{Q_L^3}\left(W_c + \frac{W_T}{\rho c_p}\right)^2(c'_0 - c_0) \quad (30)$$

$$T_3 = T_2 + \frac{Q_{T3}}{\rho c_p Q_L} = \frac{3W_T}{Q_L c_p \rho}(T'_0 - T_0) + T_0 - \frac{3W_T}{Q_L^2 c_p \rho}\left(W_c + \frac{W_T}{c_p\rho}\right)(T'_0 - T_0) + \frac{W_T}{Q_L^3 c_p \rho}\left(W_c + \frac{W_T}{c_p\rho}\right)^2(T'_0 - T_0) \quad (31)$$

where $Q_L = A \cdot U_\infty$. The dissolved gas concentration in the nth liquid slug, temperature in the nth liquid slug, mass flux from the nth unit cell and heat flux from the nth unit cell are determined by the following formulas:

$$c_n = c_0 + \frac{\sum_{k=1}^{n} Q_{ck}}{Q_L},\ T_n = T_0 + \frac{\sum_{k=1}^{n} Q_{Tk}}{\rho c_p Q_L} \quad (32)$$

$$Q_{cn} = W_c(dT_{n-1} + b - c_{n-1}), \quad (33)$$

$$Q_{Tn} = W_T\left(\frac{c_{n-1} - b}{d} - T_{n-1}\right). \quad (34)$$

Using the above results we derived the following expressions for c_n, T_n, Q_{cn} and Q_{Tn} in gas–liquid slug flow:

$$c_n = W_c(c'_0 - c_0)\sum_{k=1}^{n}\frac{1}{Q_L^k}\left(\frac{W_T}{\rho c_p} + W_c\right)^{k-1}(-1)^{k-1}\binom{n}{k}. \quad (35)$$

The coefficients in expression (35) coincide with those in the Pascal triangle. After simple algebra we obtain a formula for a dissolved gas concentration in the nth liquid plug:

$$c_n = c_0 + \frac{(c'_0 - c_0)W_c}{(W_T/c_p\rho + W_c)}\left(1 - \left(1 - \frac{1}{Q_L}\left(W_c + \frac{W_T}{c_p\rho}\right)\right)^n\right). \quad (36)$$

Formula (36) is valid for $n \geqslant 1$. From (36) we determine the dissolved gas concentration in the liquid plug for $n \to \infty$:

$$c_\infty = c_0 + \frac{(c'_0 - c_0)W_c}{W_T/c_p\rho + W_c} = \frac{dT_0 + b - c_0 K^{-1}}{1 - K^{-1}}. \quad (37)$$

For isothermal absorption (36) yields:

$$c_n = c_0 + (c_s - c_0)\left(1 - \left(1 - \frac{W_c}{Q_L}\right)^n\right) \quad (38)$$

where c_s is the concentration of saturation at gas–liquid interface. Similarly, the expression for temperature reads:

$$T_n = T_0 + \frac{W_T}{c_p\rho}(T'_0 - T_0)\sum_{k=1}^{n}\frac{1}{Q_L^k}\times\left(\frac{W_T}{\rho c_p}+W_c\right)^{k-1}(-1)^{k-1}\binom{n}{k}. \quad (39)$$

After some algebra we find

$$T_n = T_0 + \frac{W_T(T'_0 - T_0)}{c_p\rho(W_T/c_p\rho + W_c)}\times\left(1-\left(1-\frac{1}{Q_L}\left(\frac{W_T}{c\rho}+W_c\right)\right)^n\right). \quad (40)$$

Formula (40) is valid for $n \geqslant 1$. From (40) we determine temperature in a liquid plug for $n \to \infty$:

$$T_\infty = T_0 + \frac{W_T(T'_0 - T_0)}{c_p\rho(W_T/c_p\rho + W_c)} = \frac{(c_0-b)/d - T_0K}{d(1-K)}. \quad (41)$$

Combining expressions (37) and (41) yields:

$$dT_\infty + b = c_\infty. \quad (42)$$

Then

$$Q_{cn} = W_c(c'_0 - c_0)\sum_{k=0}^{n-1}\frac{1}{Q_L^k}(W_c + W_T/c_p\rho)^k(-1)^k\binom{n-1}{k} \quad (43)$$

After some algebra we find that

$$Q_{cn} = W_c(c'_0 - c_0)\left(1-\frac{1}{Q_L}(W_c+W_T/c_p\rho)\right)^{n-1}. \quad (44)$$

Formula (44) is valid for $n \geqslant 1$. Expression (44) implies that a mass flux from the unit cell with $n \to \infty$ is equal to zero ($Q_c|_{n\to\infty} = 0$). Similarly

$$Q_{Tn} = W_T(T'_0 - T_0)\sum_{k=0}^{n-1}\frac{1}{Q_L^k}\left(W_c+\frac{W_T}{c_p\rho}\right)^k(-1)^k\binom{n-1}{k}. \quad (45)$$

After some algebra we find that

$$Q_{Tn} = W_T(T'_0 - T_0)\left(1-\frac{1}{Q_L}\left(W_c+\frac{W_T}{c_p\rho}\right)\right)^{n-1}. \quad (46)$$

Formula (46) is valid for $n \geqslant 1$. Expression (46) implies that heat flux from the unit cell with $n \to \infty$ is equal to zero ($Q_T|_{n\to\infty} = 0$). The total mass flux from all N unit cells is determined by the following formula:

$$Q_{c\Sigma N} = \sum_{n=1}^{N} Q_{cn}. \quad (47)$$

Therefore,

$$Q_{c\Sigma N} = \frac{W_cQ_L(c'_0 - c_0)}{(W_c + W_T/c_p\rho)}\left(1-\left(1-\frac{1}{Q_L}(W_c+W_T/c_p\rho)\right)^N\right). \quad (48)$$

Formula (48) is valid for $N \geqslant 1$. Mass flux from an infinite cluster of gas slugs can be determined from equation (48):

$$Q_{c\Sigma\infty} = \frac{Q_LW_c(c'_0 - c_0)}{W_T/c_p\rho + W_c} = Q_L\cdot(c_\infty - c_0) \quad (49)$$

where c_∞ is determined by expression (37). Ratio of the mass flux from the nth unit cell of slug flow to a total mass flux from a cluster with an infinite number of unit cells is determined by the following formula:

$$\frac{Q_{cn}}{Q_{c\Sigma\infty}} = M(1-M)^{n-1} \quad (50)$$

where $M = (W_T/c_p\rho + W_c)/Q_L$. The ratio of total mass flux from a cluster with N unit cells to the mass flux from a cluster with an infinite number of unit cells is determined by the following formula:

$$\frac{Q_{c\Sigma N}}{Q_{c\Sigma\infty}} = 1-(1-M)^N. \quad (51)$$

The total heat flux from N unit cells can be determined as follows:

$$Q_{T\Sigma N} = \sum_{n=1}^{N} Q_{Tn}.$$

After some algebra the latter expression yields

$$Q_{T\Sigma N} = \frac{W_TQ_\ell(T'_0 - T_0)}{(W_c + W_T/c_p\rho)}\left(1-\left(1-\frac{1}{Q_\ell}(W_c+W_T/c_p\rho)\right)^N\right). \quad (52)$$

Formula (52) is valid for $N \geqslant 1$. Heat flux from an infinite cluster of Taylor bubbles is

$$Q_{T\Sigma\infty} = \frac{W_TQ_L(T'_0 - T_0)}{W_c + W_T/c_p\rho} = Q_L(T_\infty - T_0)c_p\rho \quad (53)$$

where T_∞ is determined by formula (41). The ratio of the heat flux from the n-th unit cell of a gas–liquid slug flow to a total heat flux from a cluster with an infinite number of unit cells is determined by the following formula:

$$\frac{Q_{Tn}}{Q_{T\Sigma\infty}} = M(1-M)^{n-1}. \quad (54)$$

The ratio of total heat flux from a cluster with N unit cells to the heat flux from a cluster with an infinite number of unit cells is determined by the following formula:

$$\frac{Q_{T\Sigma N}}{Q_{T\Sigma\infty}} = 1-(1-M)^N. \quad (55)$$

The above approach is valid if $M < 1$. Following Taitel et al. [36] we assume that slug flow is stable when $L_L = 8d_{ch}$. Dependence of the normalized mass flux $Q_{cn}/Q_{c\Sigma\infty}$ on dimensionless number $(W_c + W_T/c_p\rho)/Q_L$ for different numbers of unit cells N is determined by formula (50) and is shown at Fig. 3. Dependence of the normalized mass flux $Q_{c\Sigma N}/Q_{c\Sigma\infty}$ upon the dimensionless number $(W_c + W_T/c_p\rho)/Q_L$ for different numbers of unit cells N is determined by formula (51) and is shown in Fig. 4. From equation (51) we derived a formula for a number of unit

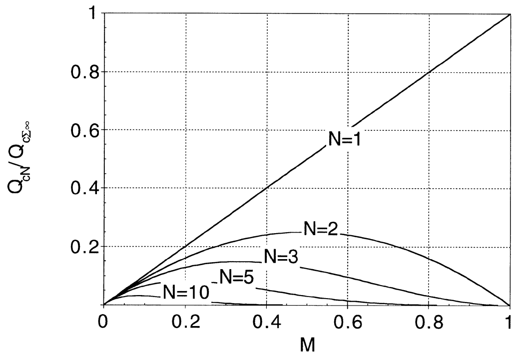

Fig. 3. Dependence of the relative mass flux from the Nth unit cell of a slug flow $Q_{cN}/Q_{c\Sigma\infty}$ vs dimensionless rate of absorption M.

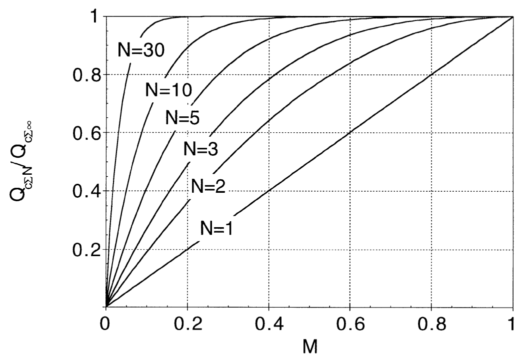

Fig. 4. Dependence of the relative mass flux from a cluster with N unit cells $Q_{c\Sigma N}/Q_{c\Sigma\infty}$ vs dimensionless rate of absorption M.

cells which contribute a given fraction $Q_{c\Sigma N}/Q_{c\Sigma\infty}$ to the total mass flux during nonisothermal absorption in an infinite cluster of unit cells:

$$N = E(1+\ln(1-Q_{c\Sigma N}/Q_{c\Sigma\infty})/\ln(1-M)) \quad (56)$$

where E is an integer part of the number function. Equation (56) implies that if $M = 0.2875$ the rate of gas absorption in the first $N = 10$ unit cells is equal to 95% of the total gas absorption rate in an infinite cluster of unit cells. The latter allows to determine an optimal length of a tube of absorber operating in a slug regime. The length of a tube must be long enough so N Taylor bubbles pass through a certain cross-section of a liquid plug during the residence time of liquid plug in a tube. Velocity of a Taylor bubble motion in a tube reads (see [38]):

$$U_{GB} = C_1\bar{U}_{LS}+U_{\infty} \quad (57)$$

where $C_1 = 2.0$ for laminar fluid flow and $C_1 = 1.2$ for turbulent flow of fluid. The time required for N Taylor bubbles to pass through a cross-section of a flowing liquid plug is:

$$t_0 = \frac{NL_c}{U_{GB}-\bar{U}_{LS}}. \quad (58)$$

Thus, the optimal length of a tube of absorber operating in a slug regime is determined by the following formula:

$$L_T = \bar{U}_{LS}t_0 = \frac{NL_c\bar{U}_{LS}}{U_{GB}-\bar{U}_{LS}}. \quad (59)$$

The ratio of the total mass flux from N unit cells without small gas bubbles to the total mass flux from N unit cells with spherical gas bubbles is determined by the following formula:

$$\frac{Q_{c\Sigma N0}}{Q_{c\Sigma N}} = \frac{1-\left(1-\frac{S}{Q_L}\left(\frac{\alpha}{c_p\rho}+\beta\right)\right)^N}{1-(1-M)^N}. \quad (60)$$

Dependence of the normalized mass flux $Q_{c\Sigma N0}/Q_{c\Sigma N}$ from the N unit cells given by formula (60) is shown in Fig. 5 for SO_2–water slug flow in a 6 mm diameter channel for $L_L = 8d_{ch}$ $L_B = 2d_{ch}$ and $\varphi_S = 0.25$. In the limiting case of an isothermal absorption without heat release expressions (36), (44) and (48) recover the formulas for isothermal absorption in a slug flow. In the limiting case without small bubbles in the liquid plugs the derived formulas for mass and heat transfer during gas absorption in slug flow recover the obtained expressions for mass and heat transfer in slug flow without small bubbles in the liquid plugs (see [7, 8]).

6. Comparison with the experimental results

Volumetric coefficient of mass transfer measured in experiments is determined by the following formula (see, e.g., [39]):

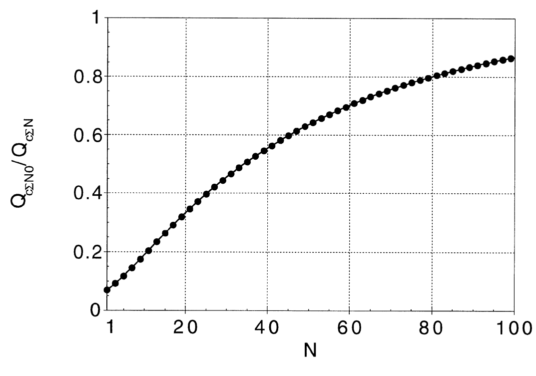

Fig. 5. Dependence of the ratio of the total mass flux from N unit cells of gas–liquid slug flow without small gas bubbles to the total mass flux from N unit cells with gas bubbles $Q_{c\Sigma N0}/Q_{c\Sigma N}$ vs number of unit cells N.

$$k_L a = \frac{U_L^s}{L_T} \ln\left(\frac{c_s - c_0}{c_s - c_e}\right) \tag{61}$$

where c_e is the concentration of dissolved gas at the exit of a tube, c_0 the concentration of dissolved gas at the entrance of a tube. In our case $c_e = c_n$, where c_n is a concentration of dissolved gas in the nth liquid plug. Substitution of (38) to (61) yields:

$$k_L a = \frac{U_L^s n}{L_T} \ln\left(1 - \frac{W_c}{Q_L}\right), \tag{62}$$

where n is a number of Taylor bubbles flowing through a cross section of a liquid slug during the residence time of a liquid slug in a channel

$$n = \frac{L_T(U_{GB} - \bar{U}_{LS})}{L_c \bar{U}_{LS}}. \tag{63}$$

In order to compare the theoretical results given by equations (6), (18), (21), (62) and (63) with the experimental data we expressed all unknown variables U_{GB}, $\bar{U}_{LS}$, L_c, φ_S through the variables U_L^s and U_G^s which are measured in experiment following the approach suggested in [6] and [38]. Superficial velocity of the two-phase mixture U_M^s is defined as (see, e.g., [38]):

$$U_M^s = U_G^s + U_L^s = \bar{U}_{GS}\varphi_S + \bar{U}_{LS}(1 - \varphi_S) \tag{64}$$

The gas and liquid flow within the liquid plug can be treated as a bubbly flow. Consequently, the absolute rise velocity of the bubble swarm in a tube and the average velocity of liquid $\bar{U}_{LS}$ are related by

$$\bar{U}_{GS} = \bar{U}_{LS} + U_b. \tag{65}$$

Combining (64) and (65) we obtain:

$$\bar{U}_{LS} = U_M^s - U_b \varphi_S \tag{66}$$

and

$$\bar{U}_{GS} = U_M^s + U_b(1 - \varphi_S). \tag{67}$$

The absolute rise velocity of the Taylor bubble in a slug flow is determined by equation (57). Combining (57) and (66) we obtain:

$$U_{GB} = C_1 U_M^s - C_1 U_b \varphi_S + U_\infty. \tag{68}$$

The volumetric-average void fraction in the entire slug unit is defined as a ratio of the gas volume in the Taylor bubble and the liquid plug section to the slug unit volume:

$$\varphi_T = \varphi_B \frac{L_B}{L_c} + \varphi_S\left(1 - \frac{L_B}{L_c}\right). \tag{69}$$

Taking into account that (see [38])

$$U_G^s = U_{GB}\varphi_B \frac{L_B}{L_c} + \bar{U}_{GS}\varphi_S\left(1 - \frac{L_B}{L_c}\right) \tag{70}$$

we obtain a general relationship for the lengths ratio L_B/L_c:

$$\frac{L_B}{L_c} = \frac{U_G^s - \bar{U}_{GS}\varphi_S}{U_{GB}\varphi_B - \bar{U}_{GS}\varphi_S}. \tag{71}$$

Equation (71) yields the lengths ratios L_c/L_L and L_B/L_L:

$$\frac{L_c}{L_L} = \frac{U_{GB}\varphi_B - \bar{U}_{GS}\varphi_S}{U_{GB}\varphi_B - U_G^s} \tag{72}$$

$$\frac{L_B}{L_L} = \frac{U_G^s - \bar{U}_{GS}\varphi_S}{U_{GB}\varphi_B - U_G^s}. \tag{73}$$

Equations (66) and (68) yield

$$U_{GB} - \bar{U}_{LS} = (C_1 - 1)(U_M^s - U_b\varphi_S) + U_\infty. \tag{74}$$

Substituting equations (66), (72), (73), and (74) to equation (62) we obtain the following expression for the volumetric coefficient of mass transfer:

$$k_L a = -\frac{U_L^s[(C_1 - 1)(U_M^s - U_b\varphi_S) + U_\infty]}{(U_M^s - U_b\varphi_S)L_L(U_{GB}\varphi_B - \bar{U}_{GS}\varphi_S)} \times \ln\left(1 - \frac{\beta_b k S_b}{Q_L} - \frac{\beta k S}{Q_L}\right) \tag{75}$$

where

$$\frac{\beta_b k S_b}{Q_L} = \frac{12\varphi_S L_L}{d_b \pi^{1/2} U_\infty}\sqrt{\frac{D U_b}{d_b}}, \tag{76}$$

$$\frac{\beta S}{Q_L} = \frac{2.52 D^{1/2} g^{1/3} L_B^{1/2}}{\pi^{1/2} \nu^{1/6} R^{1/2} U_\infty} \tag{77}$$

and U_{GB}, $\bar{U}_{GS}$ and U_M^s are expressed through U_L^s and U_G^s by equations (64), (67) and (68). Equation (77) is derived in the approximation that the thickness of a liquid film between the Taylor bubble and a wall is determined by (see, e.g., [40]) $\delta = 0.9R^{1/2}\nu^{1/3}g^{1/6}$ and $u = Q_L/(2\pi R\delta)$. Following [36] and [41] we assume that $L_L = 8d$ and $\varphi_B = 0.85$. The dependence of the gas content in a liquid plug on gas and liquid superficial velocities is determined by the following equation suggested in [41]:

$$\varphi_S = \frac{U_G^s}{C_2 + C_3(U_G^s + U_L^s)} \tag{78}$$

where $C_2 = 0.033$ and $C_3 = 1.25$. Dependence of the void fraction in the liquid slug on gas superficial velocity for different values of liquid superficial velocity is shown in Fig. 6. In the experimental study [42] the volumetric mass transfer coefficients were measured in a vertical channel with a cocurrent upward two-phase flow, and oxygen absorption by deionized water in a channel with 1.9 cm internal diameter was investigated. Theoretical results given by equation (75) were compared with experimental data [42] corresponding to a fully developed slug flow (see Figs 7–9). In experiments in [42] a slug flow was observed in the range of gas superficial velocities from 0.1 m s^{-1} up to 0.6 m s^{-1} and volumetric mass transfer coefficient was measured for the values of liquid superficial velocities equal to 0.24, 1.0 and 1.49 m s^{-1}.

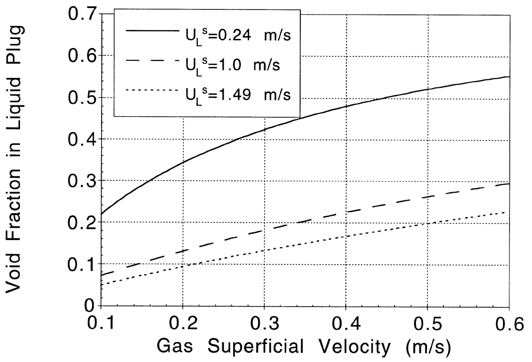

Fig. 6. Dependence of void fraction in liquid plug vs gas superficial velocity.

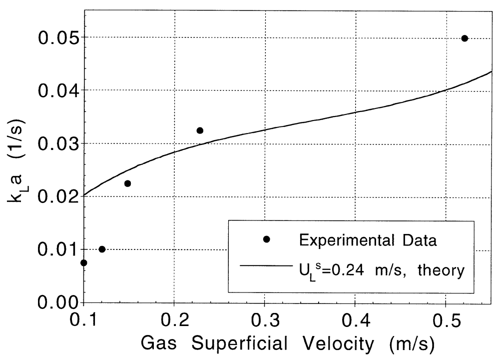

Fig. 7. Dependence of volumetric mass transfer coefficient vs gas superficial velocity. $U_L^s = 0.24$ m s^{-1}.

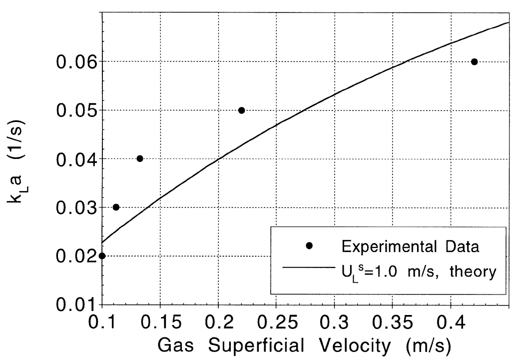

Fig. 8. Dependence of volumetric mass transfer coefficient vs gas superficial velocity. $U_L^s = 1.0$ m s^{-1}.

7. Discussion and conclusions

Results presented in Fig. 3 show that mass flux in gas–liquid slug flow decreases when the unit cell number increases. Thus, e.g., for $M = 0.4$ the contribution of the

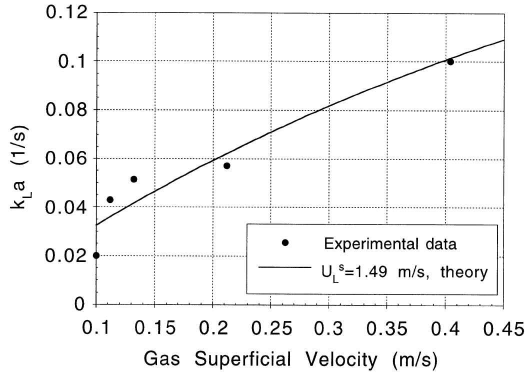

Fig. 9. Dependence of volumetric mass transfer coefficient vs gas superficial velocity. $U_L^s = 1.49$ m s^{-1}.

second unit cell is equal to 30% of the total mass flux in gas–liquid slug flow with an infinite number of slugs, and the contribution of the 10th cell amounts to less than 1%. In spite of the fact that the coefficient of mass transfer is the same for all Taylor bubbles and for all spherical gas bubbles, mass fluxes from different Taylor bubbles and small spherical gas bubbles in different liquid plugs differs due to the increase of the concentration of the dissolved gas in the consecutive liquid plugs. Figure 4 shows that the contribution of the first 10 unit cells of a gas–liquid slug flow for $M = 0.2$ amounts to 90%. Results presented in Fig. 5 show that the normalized mass flux $Q_{c\Sigma N0}/Q_{c\Sigma N}$ tends to unity with increase of a number of unit cells.

We developed a model for the analysis of combined mass and heat transfer during nonisothermal absorption in a vertical gas–liquid slug flow and derived simple expressions for mass and heat transfer rates from a single Taylor bubble and in a gas–liquid slug flow. In the limiting case without heat release the derived expressions recover formulas for isothermal absorption from a single Taylor bubble and in gas–liquid slug flow. In the case without small bubbles in the liquid plugs the derived formulas for mass and heat transfer during gas absorption in a slug flow recover the previously obtained expressions for mass and heat transfer in slug flow without small bubbles in the liquid plugs. Both experimental and theoretical results show increase of $k_L a$ when gas superficial velocity U_G^s and liquid superficial velocity U_L^s increase. In accordance with equation (75), a contribution of small spherical bubbles in the liquid plug to mass transfer is considerably higher than the contribution of a Taylor bubble. In accordance with equation (78) suggested in [41] and Fig. 6 void fraction in the liquid plug increases when gas superficial velocity increases. Increase of a void fraction in the liquid plug causes increase of a number of spherical gas bubbles in the liquid plug. Thus the volumetric mass transfer coefficient for gas–liquid slug flow increases when gas superficial velocity increases.

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